

Prof: Julio César Alonso C

**MAS sin reposición**

$$\frac{n}{N} \quad \frac{N!}{n!(N-n)!}$$

$$S_{\bar{y}} = \sqrt{\left(1 - \frac{n}{N}\right)} \frac{S}{\sqrt{n}}$$

$$\hat{P} = \frac{\sum_{i=1}^n X_i}{n} \quad S_{\hat{P}}^2 = \frac{N-n}{N-1} \frac{\hat{P}\hat{Q}}{n}$$

**MAS con reposición**

$$1 - \left(\frac{N-1}{N}\right)^n \quad N^n$$

$$S_{\bar{y}} = \frac{S}{\sqrt{n}}$$

$$S_{\hat{P}}^2 = \frac{\hat{P}\hat{Q}}{n}$$

**MEA**

$$W_h = \frac{N_h}{N} \text{ para } h = 1, 2, \dots, H$$

$$n = \frac{\frac{z_{\alpha/2}^2 S^2}{\delta^2}}{1 + \frac{1}{N} \left( \frac{z_{\alpha/2}^2 S^2}{\delta^2} \right)}$$

$$n_{(p)} = \frac{\frac{z_{\alpha/2}^2 \hat{P}\hat{Q}}{\delta^2}}{\frac{N-1}{N} + \frac{1}{N} \left( \frac{z_{\alpha/2}^2 \hat{P}\hat{Q}}{\delta^2} \right)}$$

$$\frac{1}{N^n}$$

$$n_0 = \frac{\frac{z_{\alpha}^2 S^2}{2}}{\delta^2}$$

$$n_{(p)} = \frac{z_{\alpha/2}^2 \hat{P}\hat{Q}}{\delta^2}$$

$$n = \sum_{h=1}^H n_h$$

$$\bar{y} = \sum_{h=1}^H W_h \bar{y}_h \quad \bar{y}_h = \frac{\sum_{i=1}^{n_h} y_{h,i}}{n_h}$$

$$S_{\bar{y}_h} = \sqrt{\left(1 - \frac{n_h}{N_h}\right)} \frac{S_h}{\sqrt{n_h}}$$

$$n_h = n \frac{W_h S_H}{\sum_{h=1}^H W_h S_H}$$

**Muestreo por Conglomerado**

$$\bar{y}_{congl} = \frac{\sum_{i=1}^{n_c} \sum_{j=1}^M y_{i,j}}{n_c}$$

$$S_{congl}^2 = \frac{\sum_{i=1}^{n_c} \left( \left( \sum_{j=1}^M y_{i,j} \right) - \bar{y}_{congl} \right)^2}{n_c - 1}$$

$$S_{\bar{y}_{congl}}^2 = \frac{N_c (N_c - n_c)}{n_c} S_{congl}^2$$

$$Var[\bar{y}] = \sum_{h=1}^H W_h^2 Var[\bar{y}_h]$$

$$n = \frac{\frac{z_{\alpha/2}^2 \left( \sum_{h=1}^H W_h S_H \right)^2}{\delta^2}}{1 + \frac{1}{N} \left( \frac{z_{\alpha/2}^2 \left( \sum_{h=1}^H W_h S_H \right)^2}{\delta^2} \right)}$$

$$\bar{y} = \frac{\sum_{i=1}^{n_c} \sum_{j=1}^M y_{i,j}}{n} = \frac{\bar{y}_{congl}}{M}$$

$$S^2 = \frac{\sum_{i=1}^{n_c} \sum_{j=1}^M (y_{i,j} - \bar{y})^2}{n - 1}$$

$$S_{\bar{y}}^2 = \frac{(N_c - n_c)}{N_c n_c} \frac{S_{congl}^2}{M^2}$$

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$$n_c = \frac{\frac{z_{\alpha/2}^2 S_{congl}^2}{\delta^2 M^2}}{1 + \frac{1}{N_c} \left( \frac{z_{\alpha/2}^2 S_{congl}^2}{\delta^2 M^2} \right)} n = n_c M$$

**Regresión**

$$\hat{\beta}_1 = \bar{y} - \hat{\beta}_2 \bar{x} \quad \hat{\beta}_2 = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2} = \frac{\sum_{i=1}^n y_i x_i - n\bar{y}\bar{x}}{\sum_{i=1}^n x_i^2 - n\bar{x}^2}$$

$$s^2 = \frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{n-2} = \frac{\sum_{i=1}^n \hat{\varepsilon}_i^2}{n-2}$$

$$Var[\hat{\beta}] = \begin{bmatrix} Var[\hat{\beta}_1] & Cov[\hat{\beta}_1, \hat{\beta}_2] \\ Cov[\hat{\beta}_1, \hat{\beta}_2] & Var[\hat{\beta}_2] \end{bmatrix} = \begin{bmatrix} \frac{\sum_{i=1}^n x_i^2}{n \sum_{i=1}^n (x_i - \bar{x})^2} \sigma^2 & \frac{-\bar{x}}{\sum_{i=1}^n (x_i - \bar{x})^2} \sigma^2 \\ \frac{-\bar{x}}{\sum_{i=1}^n (x_i - \bar{x})^2} \sigma^2 & \frac{1}{\sum_{i=1}^n (x_i - \bar{x})^2} \sigma^2 \end{bmatrix}$$

$$SSR = \sum_{i=1}^n (\hat{y}_i - \bar{y})^2 \quad SST = \sum_{i=1}^n (y_i - \bar{y})^2 \quad SSE = \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

$$r^2 = \frac{SSR}{SST}$$

$$\hat{y} \pm t_{\alpha/2, n-2} s \sqrt{1 + \frac{1}{n} + \frac{(x_p - \bar{x})^2}{\sum_{i=1}^n (x_i - \bar{x})^2}}$$

$$\hat{y} \pm t_{\alpha/2, n-2} s \sqrt{\frac{1}{n} + \frac{(x_p - \bar{x})^2}{\sum_{i=1}^n (x_i - \bar{x})^2}}$$

$$X^T X = \begin{bmatrix} n & \sum_{i=1}^n X_{1i} & \sum_{i=1}^n X_{2i} & \dots & \sum_{i=1}^n X_{ki} \\ \sum_{i=1}^n X_{1i}^2 & \sum_{i=1}^n X_{1i} X_{2i} & \dots & \sum_{i=1}^n X_{1i} X_{ki} \\ & \sum_{i=1}^n X_{2i}^2 & \ddots & \sum_{i=1}^n X_{2i} X_{ki} \\ & & \ddots & \vdots \\ & & & \sum_{i=1}^n X_{ki}^2 \end{bmatrix}$$

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$$X^T y = \begin{bmatrix} \sum_{i=1}^n y_i \\ \sum_{i=1}^n y_i X_{1i} \\ \sum_{i=1}^n y_i X_{2i} \\ \vdots \\ \sum_{i=1}^n y_i X_{ki} \end{bmatrix}$$

$$y^T y = \sum_{i=1}^n y_i^2$$

|            |                 |
|------------|-----------------|
| $n$        | $t_{0.01, n-2}$ |
| <b>120</b> | <b>2.357</b>    |

$$\hat{\beta} = (X^T X)^{-1} X^T y$$

$$s^2 = \frac{SSE}{n-k} = \frac{y^T y - \hat{\beta}^T X^T y}{n-k}$$

$$\hat{\beta}_i \pm t_{\frac{\alpha}{2}, n-k} s_{\hat{\beta}_i}$$

$$t = \frac{\hat{\beta}_i - c}{s_{\hat{\beta}_i}}$$

**Cantidades Importantes**

$$\sqrt{2} = 1.414 \quad \sqrt{3} = 1.732 \quad \sqrt{5} = 2.236 \quad \sqrt{7} = 2.646 \quad \sqrt{10} = 3.162$$

$$\sqrt{13} = 3.606$$

**Valores críticos de la distribución normal y t**

| $\alpha$    | $z_{\frac{\alpha}{2}}$ | $z_{\alpha}$ |
|-------------|------------------------|--------------|
| <b>0.01</b> | <b>2.58</b>            | <b>2.33</b>  |
| <b>0.05</b> | <b>1.96</b>            | <b>1.64</b>  |
| <b>0.1</b>  | <b>1.64</b>            | <b>1.28</b>  |

## Econometría, Examen Parcial II

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### Fórmulas

$$X^T X = \begin{bmatrix} n & \sum_{i=1}^n X_{1i} & \sum_{i=1}^n X_{2i} & \cdots & \sum_{i=1}^n X_{ki} \\ \sum_{i=1}^n X_{1i}^2 & \sum_{i=1}^n X_{1i} X_{2i} & \cdots & \sum_{i=1}^n X_{1i} X_{ki} \\ \sum_{i=1}^n X_{2i}^2 & \ddots & \sum_{i=1}^n X_{2i} X_{ki} \\ \vdots & \vdots & \vdots & \vdots \\ \sum_{i=1}^n X_{ki}^2 \end{bmatrix}$$

$$X^T y = \begin{bmatrix} \sum_{i=1}^n y_i \\ \sum_{i=1}^n y_i X_{1i} \\ \sum_{i=1}^n y_i X_{2i} \\ \vdots \\ \sum_{i=1}^n y_i X_{ki} \end{bmatrix}$$

$$y^T y = \sum_{i=1}^n y_i^2$$

$$SST = \sum_{i=1}^n (y_i - \bar{y})^2 = y^T y - n\bar{y}^2$$

$$SSE = \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

$$\hat{\beta} = (X^T X)^{-1} X^T y$$

$$s^2 = \frac{SSE}{n-k} = \frac{y^T y - \hat{\beta}^T X^T y}{n-k}$$

$$Var[\hat{\beta}] = \sigma^2 (X^T X)^{-1}$$

$$SSR = \sum_{i=1}^n (\hat{y}_i - \bar{y})^2 = \hat{\beta}^T X^T y - n\bar{y}^2 \quad t = \frac{\hat{\beta}_i - c}{s_{\hat{\beta}_i}}$$

$$F_c = \frac{(c - R\hat{\beta})^T (R(X^T X)^{-1} R^T)^{-1} (c - R\hat{\beta}) / r}{SSE / (n-k)}$$

$$F_C = \frac{R^2 / (k-1)}{(1-R^2) / (n-k)} = \frac{MSR}{MSE}$$

$$F_c = \frac{(SSE_R - SSE_U) / r}{SSE_U / (n-k)} \quad R^2 = \frac{SSR}{SST}$$

$$\hat{\beta}_i \pm t_{\frac{\alpha}{2}, n-k} s_{\hat{\beta}_i} \quad \bar{R}^2 = 1 - (1-R^2) \frac{n-1}{n-k}$$

$$\hat{y}_p = x_p^T \hat{\beta}, \quad x_p^T = (1 \quad x_{1p} \quad x_{2p} \quad \cdots \quad x_{kp})$$

$$\hat{y}_p \pm t_{\frac{\alpha}{2}, n-k} \sqrt{\sigma^2 x_p^T (X^T X)^{-1} x_p}$$

$$\hat{y}_p \pm t_{\frac{\alpha}{2}, n-k} \sqrt{\sigma^2 \left[ 1 + x_p^T (X^T X)^{-1} x_p \right]}$$

$$\hat{\beta}_j^E = \hat{\beta}_j \frac{s_{X_j}}{s_y}, \quad j = 2, 3, \dots, k \quad E_j = \hat{\beta}_j \frac{\bar{X}_j}{\bar{y}}$$

## Econometría, Examen Parcial II

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### Cantidades Importantes

#### Test de Heteroscedasticidad

**Goldfeld y Quand:**  $F_{GQ} = \frac{SSE_2}{SSE_1} \sim F_{(n-d-2k, n-d-2k)}$

**Breush-Pagan:**  $\frac{\hat{\varepsilon}_i^2}{\hat{\sigma}^2} = \gamma + \delta Z_i + \mu_i$ ,  $BP = \frac{SSR}{2} \sim \chi_g^2$

**White:**  $\hat{\varepsilon}_i^2 = \gamma + \sum_{m=1}^k \sum_{j=1}^k \delta_s X_{mi} X_{ji} + \mu_i$ ,  $W_a = nR^2 \sim \chi_g^2$

#### Test de Autocorrelación

**Durbin-Watson**  $DW \approx 2(1 - \hat{\rho})$

| <b>Ho</b>        | <b>Sí</b>            | <b>Decisión</b> |
|------------------|----------------------|-----------------|
| $H_0 : \rho = 0$ | $d_u < DW < 4 - d_u$ | <b>A</b>        |
| <b>No auto +</b> | $0 < DW < d_l$       | <b>R</b>        |
| <b>No auto -</b> | $4 - d_l < DW < 4$   | <b>R</b>        |

**Área de indecisión**  $d_l < DW < d_u$  y  $4 - d_u < DW < 4 - d_l$

$$\sqrt{2} = 1.414$$

$$\sqrt{10} = 3.162$$

$$\sqrt{3} = 1.732$$

$$\sqrt{13} = 3.606$$

***$d_l$  y  $d_u$  para el test de DW al nivel de significancia del 5%***

| <b>N</b>   | <b>k-1=1</b>            |                         | <b>k-1=2</b>            |                         | <b>k-1=3</b>            |                         |
|------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|
|            | <b><math>d_l</math></b> | <b><math>d_u</math></b> | <b><math>d_l</math></b> | <b><math>d_u</math></b> | <b><math>d_l</math></b> | <b><math>d_u</math></b> |
| <b>50</b>  | <b>1.50</b>             | <b>1.59</b>             | <b>1.46</b>             | <b>1.63</b>             | <b>1.42</b>             | <b>1.67</b>             |
| <b>60</b>  | <b>1.55</b>             | <b>1.62</b>             | <b>1.51</b>             | <b>1.65</b>             | <b>1.48</b>             | <b>1.69</b>             |
| <b>95</b>  | <b>1.64</b>             | <b>1.69</b>             | <b>1.62</b>             | <b>1.71</b>             | <b>1.60</b>             | <b>1.73</b>             |
| <b>100</b> | <b>1.65</b>             | <b>1.69</b>             | <b>1.63</b>             | <b>1.72</b>             | <b>1.61</b>             | <b>1.74</b>             |

#### Condición de Orden

$k_i > g_i - 1$  sobre-identificada

$k_i = g_i - 1$  perfectamente identificada

**MAS sin reposición**

$$\frac{n}{N} \quad \frac{N!}{n!(N-n)!}$$

$$S_{\bar{y}} = \sqrt{\left(1 - \frac{n}{N}\right)} \frac{S}{\sqrt{n}}$$

$$\hat{P} = \frac{\sum_{i=1}^n X_i}{n} \quad S_{\hat{P}}^2 = \frac{N-n}{N-1} \frac{\hat{P}\hat{Q}}{n}$$

**MAS con reposición**

$$1 - \left(\frac{N-1}{N}\right)^n \quad N^n$$

$$S_{\bar{y}} = \frac{S}{\sqrt{n}}$$

$$S_{\hat{P}}^2 = \frac{\hat{P}\hat{Q}}{n}$$

**MEA**

$$W_h = \frac{N_h}{N} \text{ para } h = 1, 2, \dots, H$$

$$n = \frac{\frac{z_{\alpha/2}^2 S^2}{\delta^2}}{1 + \frac{1}{N} \left( \frac{z_{\alpha/2}^2 S^2}{\delta^2} \right)}$$

$$n_{(p)} = \frac{\frac{z_{\alpha/2}^2 \hat{P}\hat{Q}}{\delta^2}}{\frac{N-1}{N} + \frac{1}{N} \left( \frac{z_{\alpha/2}^2 \hat{P}\hat{Q}}{\delta^2} \right)}$$

$$\frac{1}{N^n}$$

$$n_0 = \frac{z_{\alpha}^2 S^2}{\delta^2}$$

$$n_{(p)} = \frac{z_{\alpha/2}^2 \hat{P}\hat{Q}}{\delta^2}$$

$$n = \sum_{h=1}^H n_h$$

$$\bar{y} = \sum_{h=1}^H W_h \bar{y}_h \quad \bar{y}_h = \frac{\sum_{i=1}^{n_h} y_{h,i}}{n_h}$$

$$S_{\bar{y}_h} = \sqrt{\left(1 - \frac{n_h}{N_h}\right)} \frac{S_h}{\sqrt{n_h}}$$

$$n_h = n \frac{W_h S_H}{\sum_{h=1}^H W_h S_H}$$

**Muestreo por Conglomerado**

$$\bar{y}_{congl} = \frac{\sum_{i=1}^{n_c} \sum_{j=1}^M y_{i,j}}{n_c}$$

$$S_{congl}^2 = \frac{\sum_{i=1}^{n_c} \left( \left( \sum_{j=1}^M y_{i,j} \right) - \bar{y}_{congl} \right)^2}{n_c - 1}$$

$$S_{\bar{y}_{congl}}^2 = \frac{N_C (N_C - n_C)}{n_C} S_{congl}^2$$

$$Var[\bar{y}] = \sum_{h=1}^H W_h^2 Var[\bar{y}_h]$$

$$n = \frac{\frac{z_{\alpha/2}^2 \left( \sum_{h=1}^H W_h S_H \right)^2}{\delta^2}}{1 + \frac{1}{N} \left( \frac{z_{\alpha/2}^2 \left( \sum_{h=1}^H W_h S_H \right)^2}{\delta^2} \right)}$$

$$\bar{y} = \frac{\sum_{i=1}^{n_c} \sum_{j=1}^M y_{i,j}}{n} = \frac{\bar{y}_{congl}}{M}$$

$$S^2 = \frac{\sum_{i=1}^{n_c} \sum_{j=1}^M (y_{i,j} - \bar{y})^2}{n - 1}$$

$$S_{\bar{y}}^2 = \frac{(N_C - n_C) S_{congl}^2}{N_C n_C M^2}$$

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$$n_c = \frac{\frac{z_{\alpha/2}^2 S_{congl}^2}{\delta^2 M^2}}{1 + \frac{1}{N_c} \left( \frac{z_{\alpha/2}^2 S_{congl}^2}{\delta^2 M^2} \right)} n = n_c M$$

$$X^T X = \begin{bmatrix} n & \sum_{i=1}^n X_{1i} & \sum_{i=1}^n X_{2i} & \cdots & \sum_{i=1}^n X_{ki} \\ \sum_{i=1}^n X_{1i}^2 & \sum_{i=1}^n X_{1i} X_{2i} & \cdots & \sum_{i=1}^n X_{1i} X_{ki} \\ & \sum_{i=1}^n X_{2i}^2 & \ddots & \sum_{i=1}^n X_{2i} X_{ki} \\ & & \ddots & \vdots \\ & & & \sum_{i=1}^n X_{ki}^2 \end{bmatrix}$$

$$X^T y = \begin{bmatrix} \sum_{i=1}^n y_i \\ \sum_{i=1}^n y_i X_{1i} \\ \sum_{i=1}^n y_i X_{2i} \\ \vdots \\ \sum_{i=1}^n y_i X_{ki} \end{bmatrix} \quad y^T y = \sum_{i=1}^n y_i^2$$

$$SST = \sum_{i=1}^n (y_i - \bar{y})^2 = y^T y - n\bar{y}^2 \quad SSE = \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

$$\hat{\beta} = (X^T X)^{-1} X^T y \quad s^2 = \frac{SSE}{n-k} = \frac{y^T y - \hat{\beta}^T X^T y}{n-k}$$

$$Var[\hat{\beta}] = \sigma^2 (X^T X)^{-1}$$

$$SSR = \sum_{i=1}^n (\hat{y}_i - \bar{y})^2 = \hat{\beta}^T X^T y - n\bar{y}^2 \quad t = \frac{\hat{\beta}_i - c}{s_{\hat{\beta}_i}}$$

$$F_c = \frac{(c - R\hat{\beta})^T (R(X^T X)^{-1} R^T)^{-1} (c - R\hat{\beta}) / r}{SSE / (n-k)}$$

$$F_C = \frac{R^2 / (k-1)}{(1-R^2) / (n-k)} = \frac{MSR}{MSE}$$

$$F_c = \frac{(SSE_R - SSE_U) / r}{SSE_U / (n-k)} \quad R^2 = \frac{SSR}{SST}$$

$$\hat{\beta}_i \pm t_{\frac{\alpha}{2}, n-k} s_{\hat{\beta}_i} \quad \bar{R}^2 = 1 - (1-R^2) \frac{n-1}{n-k}$$

$$\hat{y}_p = x_p^T \hat{\beta} \quad x_p^T = (1 \quad x_{1p} \quad x_{2p} \quad \dots \quad x_{kp})$$

$$\hat{y}_p \pm t_{\frac{\alpha}{2}, n-k} \sqrt{\sigma^2 x_p^T (X^T X)^{-1} x_p}$$

$$\hat{y}_p \pm t_{\frac{\alpha}{2}, n-k} \sqrt{\sigma^2 \left[ 1 + x_p^T (X^T X)^{-1} x_p \right]}$$

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$$\hat{\beta}_j^E = \hat{\beta}_j \frac{s_{x_j}}{s_y}, \quad j = 2, 3, \dots, k$$

$$E_j = \hat{\beta}_j \frac{\bar{X}_j}{\bar{y}}$$

$$h = \hat{\rho} \sqrt{\frac{n}{1 - n(\overline{\text{Var}(\hat{\alpha})})}} \quad \text{donde}$$

$$y_t = \beta_0 + \beta_1 x_{1t} + \dots + \beta_k x_{kt} + \alpha y_{t-1} + \varepsilon_t$$

**Test de Heteroscedasticidad**

**Goldfeld y Quand:**  $F_{GQ} = \frac{SSE_2}{SSE_1} \sim F_{(n-d-2k, n-d-2k)}$

**Breush-Pagan:**  $\frac{\hat{\varepsilon}_i^2}{\hat{\sigma}^2} = \gamma + \delta Z_i + \mu_i, \quad BP = \frac{SSR}{2} \sim \chi_g^2$

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**Test de Autocorrelación**

**Durbin-Watson**  $DW \approx 2(1 - \hat{\rho})$

| Ho               | Sí                   | Decisión |
|------------------|----------------------|----------|
| $H_0 : \rho = 0$ | $d_u < DW < 4 - d_u$ | <b>A</b> |
| <b>No auto +</b> | $0 < DW < d_l$       | <b>R</b> |
| <b>No auto -</b> | $4 - d_l < DW < 4$   | <b>R</b> |

Área de indecisión  $d_l < DW < d_u$  y  $4 - d_u < DW < 4 - d_l$

**d<sub>l</sub> y d<sub>u</sub> para el test de DW al nivel de significancia del 5%**

| N          | k-1=1          |                | k-1=2          |                | k-1=3          |                |
|------------|----------------|----------------|----------------|----------------|----------------|----------------|
|            | d <sub>l</sub> | d <sub>u</sub> | d <sub>l</sub> | d <sub>u</sub> | d <sub>l</sub> | d <sub>u</sub> |
| <b>50</b>  | <b>1.50</b>    | <b>1.59</b>    | <b>1.46</b>    | <b>1.63</b>    | <b>1.42</b>    | <b>1.67</b>    |
| <b>60</b>  | <b>1.55</b>    | <b>1.62</b>    | <b>1.51</b>    | <b>1.65</b>    | <b>1.48</b>    | <b>1.69</b>    |
| <b>95</b>  | <b>1.64</b>    | <b>1.69</b>    | <b>1.62</b>    | <b>1.71</b>    | <b>1.60</b>    | <b>1.73</b>    |
| <b>100</b> | <b>1.65</b>    | <b>1.69</b>    | <b>1.63</b>    | <b>1.72</b>    | <b>1.61</b>    | <b>1.74</b>    |

**Condición de Orden**

$k_i > g_i - 1$  *sobre-identificada*

$k_i = g_i - 1$  *perfectamente identificada*



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Regresión simple

$$\hat{\beta}_1 = \bar{y} - \hat{\beta}_2 \bar{x} \quad \hat{\beta}_2 = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2} = \frac{\sum_{i=1}^n y_i x_i - n\bar{y}\bar{x}}{\sum_{i=1}^n x_i^2 - n\bar{x}^2}$$

$$s^2 = \frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{n-2} = \frac{\sum_{i=1}^n \hat{\varepsilon}_i^2}{n-2}$$

$$Var[\hat{\beta}] = \begin{bmatrix} Var[\hat{\beta}_1] & Cov[\hat{\beta}_1, \hat{\beta}_2] \\ Cov[\hat{\beta}_1, \hat{\beta}_2] & Var[\hat{\beta}_2] \end{bmatrix} = \begin{bmatrix} \frac{\sum_{i=1}^n x_i^2}{n\sum_{i=1}^n (x_i - \bar{x})^2} \sigma^2 & \frac{-\bar{x}}{\sum_{i=1}^n (x_i - \bar{x})^2} \sigma^2 \\ \frac{-\bar{x}}{\sum_{i=1}^n (x_i - \bar{x})^2} \sigma^2 & \frac{1}{\sum_{i=1}^n (x_i - \bar{x})^2} \sigma^2 \end{bmatrix}$$

$$SSR = \sum_{i=1}^n (\hat{y}_i - \bar{y})^2 \quad SST = \sum_{i=1}^n (y_i - \bar{y})^2 \quad SSE = \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

$$r^2 = \frac{SSR}{SST} \quad \hat{\beta}_i \pm t_{\frac{\alpha}{2}, n-k} s_{\hat{\beta}_i} \quad t = \frac{\hat{\beta}_i - c}{s_{\hat{\beta}_i}}$$

$$\hat{y} \pm t_{\alpha/2, n-2} s \sqrt{1 + \frac{1}{n} + \frac{(x_p - \bar{x})^2}{\sum_{i=1}^n (x_i - \bar{x})^2}}$$

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Regresión múltiple

$$X^T X = \begin{bmatrix} n & \sum_{i=1}^n X_{1i} & \sum_{i=1}^n X_{2i} & \dots & \sum_{i=1}^n X_{ki} \\ \sum_{i=1}^n X_{1i} & \sum_{i=1}^n X_{1i}^2 & \sum_{i=1}^n X_{1i} X_{2i} & \dots & \sum_{i=1}^n X_{1i} X_{ki} \\ \sum_{i=1}^n X_{2i} & \sum_{i=1}^n X_{1i} X_{2i} & \sum_{i=1}^n X_{2i}^2 & \dots & \sum_{i=1}^n X_{2i} X_{ki} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \sum_{i=1}^n X_{ki} & \sum_{i=1}^n X_{1i} X_{ki} & \sum_{i=1}^n X_{2i} X_{ki} & \dots & \sum_{i=1}^n X_{ki}^2 \end{bmatrix}$$

$$X^T y = \begin{bmatrix} \sum_{i=1}^n y_i \\ \sum_{i=1}^n y_i X_{1i} \\ \sum_{i=1}^n y_i X_{2i} \\ \vdots \\ \sum_{i=1}^n y_i X_{ki} \end{bmatrix}$$

$$y^T y = \sum_{i=1}^n y_i^2$$

$$\hat{\beta} = (X^T X)^{-1} X^T y$$

$$s^2 = \frac{SSE}{n-k} = \frac{y^T y - \hat{\beta}^T X^T y}{n-k}$$

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**Cantidades Importantes**

$$\sqrt{2} = 1.414 \quad \sqrt{3} = 1.732 \quad \sqrt{5} = 2.236 \quad \sqrt{7} = 2.646 \quad \sqrt{10} = 3.162$$
$$\sqrt{13} = 3.606$$

**Valores críticos de la distribución t**

| $n$        | $t_{0.01, n-2}$ |
|------------|-----------------|
| <b>120</b> | <b>2.357</b>    |

Prof: Julio César Alonso C

Regresión simple

$$\hat{\beta}_1 = \bar{y} - \hat{\beta}_2 \bar{x} \quad \hat{\beta}_2 = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2} = \frac{\sum_{i=1}^n y_i x_i - n\bar{y}\bar{x}}{\sum_{i=1}^n x_i^2 - n\bar{x}^2}$$

$$s^2 = \frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{n-2} = \frac{\sum_{i=1}^n \hat{\varepsilon}_i^2}{n-2}$$

$$Var[\hat{\beta}] = \begin{bmatrix} Var[\hat{\beta}_1] & Cov[\hat{\beta}_1, \hat{\beta}_2] \\ Cov[\hat{\beta}_1, \hat{\beta}_2] & Var[\hat{\beta}_2] \end{bmatrix} = \begin{bmatrix} \frac{\sum_{i=1}^n x_i^2}{n \sum_{i=1}^n (x_i - \bar{x})^2} \sigma^2 & \frac{-\bar{x}}{\sum_{i=1}^n (x_i - \bar{x})^2} \sigma^2 \\ \frac{-\bar{x}}{\sum_{i=1}^n (x_i - \bar{x})^2} \sigma^2 & \frac{1}{\sum_{i=1}^n (x_i - \bar{x})^2} \sigma^2 \end{bmatrix}$$

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