MULTIPLE FACILITY LOCATION ANALYSIS PROBLEM WITH WEIGHTED EUCLIDEAN DISTANCE

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ABSTRACT

This paper presents a new graphical technique for cluster formation in multiple facilities location analysis problem with weighted Euclidean distance norm. There are two facets of the problem; location of facilities and allocation of customers. The objective is to minimize the maximum weighted distance traveled within clusters. Known parameters are Cartesian coordinates of customer locations, their demand weights, and a number of facilities to be placed. The new procedure ensures the optimum solution.

INTRODUCTION

Multiple facilities location analysis begins with formation of clusters of customers so that services can be rendered efficiently and cost effectively from facility to all customers in that set of customers. In this paper, we present new graphical technique developed to form the clusters of customers. Location of each facility can then be determined using Vector Method (Davalbhakta and Sule, 2003) developed earlier.

Multiple facilities location analysis is a long-standing topic of research and literature search in this area results in several methods and heuristics to solve different types of the problem. Brimberg *et al.* (2000) have presented extensive empirical study of improvements and comparisons of various old and recent heuristics and algorithms, which include alternative location-allocation, projection, Tabu search, *p*-median, genetic search and various versions of variable neighborhood search. This paper underlines the fact that the problem has been studied for long time and yet continues to attract the attention of researchers because of its ubiquitous nature.

The problem considered in this paper is to determine the locations of *n* facilities and to allocate the customers to them so that demands of all the customers are fulfilled through the minimum possible travel. Same problem has been addressed by Cooper (1961) where he has presented a heuristic for as many as 10 customer locations. For more customer locations, however, he has solved the mathematical equations to come up with exact solutions. Sule (2000) in his book on logistics of facilities location and allocation has also addressed the same problem and author has specified analytical method to solve the problem.

However, quite often the problem considered the travel between the new facilities in addition to that of between customers and facilities. Several examples of this type are also found during the bibliographical search (Love, Wesolowsky and Kraemer 1973), (Elzinga, Hearn, Randolph 1976), (Charalambous 1981) (Brandy, Rosenthal and Young 1983). Mathematically, the equations in this case also consider the demand weights of facilities and Euclidean distance between them in addition to the parameters presented in equation (1) later in this paper.

Other different aspects of this problem have also been studied such as minimum covering ellipse problem where a minimum certain distance has to be maintained between two facilities. Douglas and Papayanopoulos (1991) have employed interactive graphical method which produces near-optimal solutions. A decision analysis approach adopted by Current, Ratick, ReVelle (1998) demonstrates analysis of various decision making parameters that affect the total number of facilities to be placed in dynamic facility location analysis.

We have however, confined our research to the static facility location and have considered that the demands of customers in a cluster can be fulfilled by the facility located in respective cluster. During our research, we also compared our method with the work done by Levin and Ben-Israel (2002), who have surveyed and compared various techniques available to determine the facilities location and then assigning the customers to the facilities.

Remaining layout of this paper includes the problem statement and mathematical formulation followed by the illustrative example along with summary and conclusion.

PROBLEM STATEMENT

Consider *'m'* facilities are to be placed within *'n'* customers. Location of each customer is expressed as $(X_a, Y_a), (X_b, Y_b) \ldots (X_n, Y_n)$. Customers are assigned 'weights' based on their demands. Let 'W_i' be the demand weight of customer *'i'* (where i = a, b, c… n)

In case of weighted minimax facility location analysis problem, total cost is the function of demand weights of customers and distances traveled between customer locations and the facility. It is expressed as:

$$Min \sum W_i \times [(X_i - X_{nf})^2 + (Y_i - Y_{nf})^2 \quad \ldots \quad (1)$$

Where

$$X_{nf} = X - CoOrdinateOfNewFacility$$

$$Y_{nf} = Y - CoOrdinateOfNewFacility$$

METHOD

This new graphical method of cluster formation begins with plotting all the customer locations on graph followed by connecting all the customer locations on the periphery in such a way that all the customer locations lie either on the boundary or within it. The perimeter of thus formed figure is calculated and is then divided into as many fractions as the number of facilities to be placed (*m*), starting from the customer location closest to the origin (0,0) of the Cartesian coordinate system. Customer locations at the fractions are then treated as the origin for the clusters. It is likely not to have a customer location situated exactly at the position specified by the fraction value. In that case, location on the boundary closest to the fraction value is selected as the origin.

Once the origins are determined, weighted Euclidean distances of rest of the customers from origins are calculated and then based on minimum weighted Euclidean distance criterion, the clusters are formed. The formulation developed during this research work takes into account the average weighted distance of every customer location entering the cluster from every other member of the cluster. The customer locations selected in the cluster are discarded from further analysis in order to avoid any duplication.

Steps to be followed are summarized as follows:

1. Plot all the customer locations on graph
2. Connect the customers on periphery making sure that all the locations are either on boundary or within it.
3. Starting from the point closest to the origin of coordinate system (0, 0); divide the perimeter into *'m´* equal fractions.
4. Existing locations at these fractional values (division points) are termed as origins. If a division point does not coincide exactly with the location of an existing customer then the location of an existing customer closest to it on the periphery is selected as an origin. These form the initial members of the associated clusters.
5. Check each location to determine in which cluster it should be assigned. This is done by checking its average weighted Euclidean distance associated with the customers that are presently assigned to each cluster. The distance is calculated as follows:

   $$d = \frac{W_j \times \sum W_i \times d_{i-j}}{W_j + \sum W_i} \quad \ldots \quad (2)$$

   Where,

   $i = AlreadyExistingCustomerLocationsInTheCluster$

   $j = EnteringLocation$
6. Join the customer with minimum weighted distance amongst all the clusters to the associated clusters. Delete the customer from further evaluations at this stage.
7. Repeat steps 5 and 6 till all the customers are assigned to an appropriate cluster.
8. The optimum facility location in each cluster is determined by applying Vector method [6].

ILLUSTRATIVE EXAMPLE

Consider two facilities to be placed to serve 12 customers. Customer locations and their demand weights are given in Table 1.

Table 1: Customer Locations and their weights

Customer	X Coordinate	Y Coordinate	Weight
A	20	46	3.0
B	15	28	2.0
C	26	35	3.0
D	50	20	2.0
E	45	15	2.0
F	1	6	2.0
G	5	9	4.0
H	12	8	4.5
I	10	2	2.5
J	11	18	5.5
K	6	13	6.0
L	1	2	3.5

All the customer locations are plotted on graph and then customers on periphery are joined together to form a boundary as shown in Figure 1.

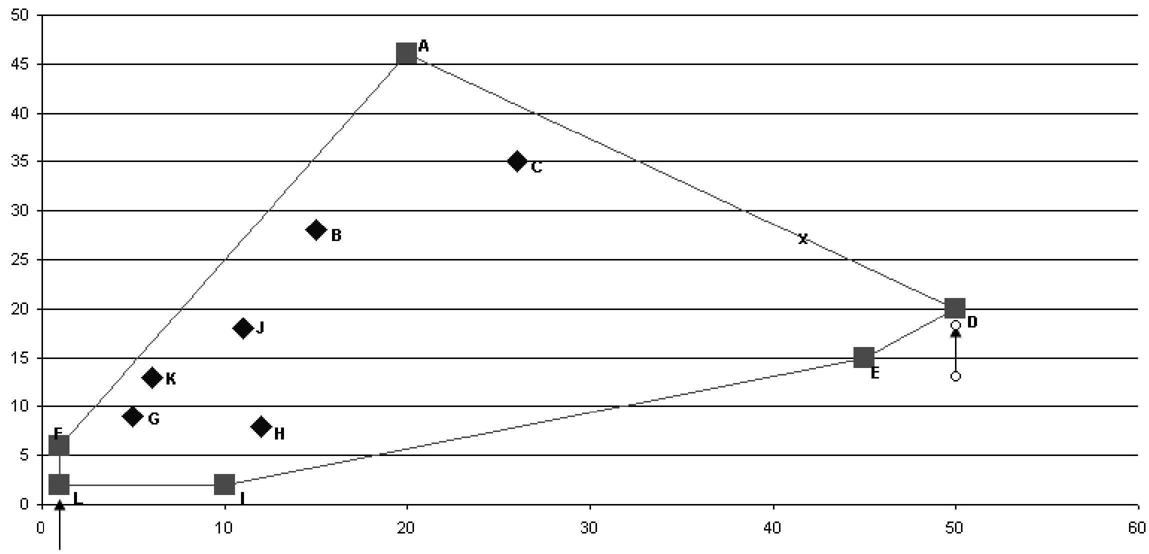

Figure 1: Customer Locations

Perimeter of the boundary is 141.3894 units. As '*m*' = 2, the fraction at which the origins should be located is 70.69471 units. First origin of the cluster is L (1, 2), while the other origin is D (50, 20), closest customer to the division point. Weighted distances are calculated using formula (2) and are shown in Table 2.

Table 2: Cluster Formation

Cycle 1: Cluster 1 Contents: L and cluster 2 contents: D

From L (1, 2): Cycle 1

Customers:	A	B	C	D	E	**F**	G	H	I	J	K	L
Weighted Distance:	77.4	37.6	66.9	66.4	58.4	**5.1**	15.0	24.7	13.1	40.4	26.7	--

From D (50, 20): Cycle 1

Customers:	A	B	C	D	E	F	G	H	I	J	K	L
Weighted Distance:	47.6	35.9	33.9	--	7.1	50.9	61.8	55.2	48.7	57.3	66.8	66.4

Customer location F is selected as the location entering the cluster originated from L.

Cycle 2: Cluster 1 Contents: L & F and cluster 2 contents: D

Cluster 1 calculations:

Customers:	A	B	C	E	G	H	I	J	K
Weighted Distance:	90.5	41.5	78.2	66.7	16.1	29.8	15.9	48.6	31.0

Cluster 2 calculations:

Customers:	A	B	C	D	**E**	G	H	I	J	K
Weighted Distance:	47.6	35.9	33.9	--	**7.1**	61.8	55.2	48.7	57.3	66.8

Customer location E is selected as a location entering cluster originated from D.

Cycle 3: Cluster 1 Contents: L & F and cluster 2 contents: D & E

Continuing the calculations in the similar manner the final cluster formation is obtained as follows:

Table 4: Resulting Clusters

Cluster 1			
Customers	**X Coordinate**	**Y Coordinate**	**Weight**
B	15	28	2.0
F	1	6	2
G	5	9	4
H	12	8	4.5
I	10	2	2.5
J	11	18	5.5
K	6	13	6
L	1	2	3.5
Cluster 2			
A	20	46	3.0
C	26	35	3.0
D	50	20	2.0
E	45	15	2.0

Cluster formation is shown in the Figure 2.

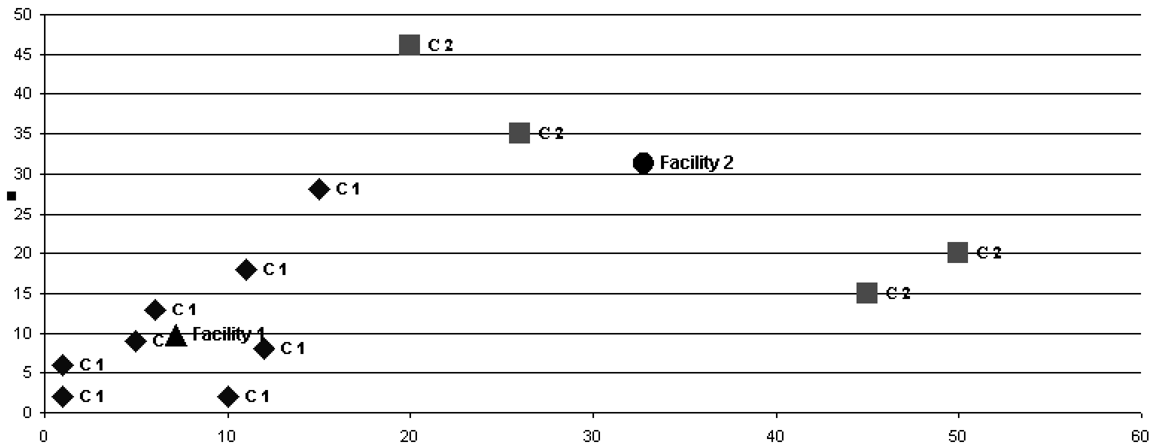

Figure 2: Cluster Formation

Facilities location analysis in each cluster is performed using the method suggested earlier which is based on vector algebra (Davalbhakta and Sule, 2003). Subsequently, facility location for cluster 1 is at (7.68, 10.96) with the optimum cost of $ 211.52. While for cluster 2, the optimum facility location is at (32.81, 31.29) with the cost of $163.61.

CONCLUSION

New graphical method has been devised to form the clusters of customer locations in a multiple facility location analysis problem with weighted minimax Euclidean distance. When we compared our method with the work done by Levin and Ben-Israel, it occurred that both the methods present the same results for the problem illustrated in their paper. However, Levin and Ben-Israel have considered that all the customers have equal demands and hence have not considered the demand weights in their heuristic. On the other hand, method proposed in this paper takes into account different demand weights of customers as vital factor in cluster formation as not only the distance between the facility and customer but the demand of customer also contributes to the total cost.

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