Cournot's Model Applied to Cell Phone Service in Colombia

Summary: Cournot's Model and Vector Error Correction are applied to the market for cell phone service in Colombia. Results indicate that the firms operating in the Eastern Zone of the country are in long-term equilibria. We find that the equilibrium adjustment velocities are statistically significant and the firms operating in the cell phone market have constant marginal costs.

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1- Introduction

In 1994 the Colombian Government divided the country's cell phone market into the Eastern, Western and Coastal Zones. In each of these zones, two cell phone providers were to compete with each other.

Market equilibria in each of the three zones are analyzed by means of Cournot's Model. Since the series have unit roots then Vectors Error Correction are used. The long-term equilibria and adjustment velocities are calculated by Johansen's method. According to Fisher (1961), the use of adjustment velocities in Cournot's Model is justified since "… no seller is ever assumed to look once and for all at other outputs and then never to look again." The results show that the adjustment velocities are statistically significant and there are constant marginal costs for Comcel and Celumovil in the Eastern Zone.

This paper is divided into five parts. Part Two analyzes the stability in each one of the three zones. Part Three discusses equilibrium in Cournot's Model through Vector Autorregression (VAR), and the Vector Error Correction (VEC) is derived. Part Four is dedicated to an empirical analysis, and in Part Five conclusions are presented.

2- The Colombian Cell Phone Market

Colombian cell phone service began in 1994 through public bidding. The objective was to establish two service providers in each zone so as to assure competition, rapid growth and lower prices to the consumer. The state-run monopoly in the telecommunications sector was thus brought to an end (Rumie, 2000).

Comcel and Celumóvil (BellSouth Oriente) were to compete in the Eastern Zone, while Celcaribe and Celumóvil de la Costa (BellSouth Costa) were to compete in the Coastal Zone. Lastly, Occel and Cocelco (BellSouth Occidente) were to compete in the Western Zone [1].

In order to explore the market behavior we analize the stability markets with the Haymer and Pashigian (1962) Instability Index (III):

$$III = \sum_{i=1}^{n} \left| \frac{S_{i,z,t} - S_{i,z,t-1}}{S_{i,z,t}} \right| \quad \forall i = \ firm, \quad z = zone, \quad t = time \qquad (1)$$

Where $S_{i,z,t}$ is the market share of company i in zone z at time t. $S_{i,z,t}$ is defined as the number of cell phone minutes sold by the firms in any given quarter. This index is developed from the absolute value of the change in market share of each service provider. It is used to indicate both positive and negative changes in market share, since both are indicative of market instability (Heggestad and Rhoades, 1976). The range of this index is from zero to one. A value of zero indicates that the market share of the firm is unchanged from the previous period, while a value of one indicates that the firms lost market share in the previous period.

The stability index in each zone may be seen in the following graph:

[insert figure 1]

As you can see the III is above zero during the entire year in the Eastern Zone. In some cases firms take two or even three quarters to recover their position. The longest recovery time registered was from the first quarter of 1997 to the second quarter of 1999. Although the index has at times strayed from zero, in general the firms have confirmed stability by maintaining their respective market shares. In the Western Zone, the III show an increasingly unstable market since 1996. In the Coastal Zone, the III indicates that the firms does not maintain their market share throughout the year. In certain cases the firms are seen to require five quarters to recover market share (1996:1 to 1997:3), while in other cases they have recovered it within only three quarters (1997:3 to 1998:2). The value of 0.86 in the third quarter of 1998 is due to Celumovil Costa's drop in market share from 63% to 42%. One can also observe that as of the second quarter of 1998, the III shows an increasing tendency to stray from zero.

3- Stochastic Version of the Duopoly Cournot Model

Let q^i_t stand for the number of minutes of cell phone communications offered by service provider i during quarter t. Let P(Q) stand for the inverse demand function (P(Q): $\Re_+ \rightarrow \Re_+$ is C^2). Let us further assume that C(Q) is the cost function (C(Q): $\Re_+ \rightarrow \Re_+$ is C^2) and P(0) > C´(0). The cost function is $c_i(q_i) = g_i + c_i q_i + 1/2\, d q_i^{\,2}$. Where $c_i > 0$, $g_i \geq 0$ and if d > 0 there are exist increasing marginal cost, If d < 0 there are decreasing marginal cost and if d = 0 there are constant marginal cost. Each seller expects that the output of every other seller will not change from t - 1 to t. On this assumption, he expects p_t to be a function of this output alone in the t period (Fisher, 1961). Then the price is $p^i_t = a - bq^i_t - b\, q^j_{t-1}$. The profit function is given by Π^i_t $(q_i,q_j) = q^i_t\,[\,a - bq^i_t - b\, q^j_{t-1}\,] - g - c_i q_i - 1/2\, dq^2_i$. And the profit maximization under incomplete information is:

$$\frac{\partial \Pi^1_t}{\partial q^1_t} = a - 2bq^1_t - bq^2_{t-1} - c_1 - dq^1_t + \varepsilon^1_t$$
$$\frac{\partial \Pi^2_t}{\partial q^2_t} = a - 2bq^2_t - bq^1_{t-1} - c_2 - dq^2_t + \varepsilon^2_t \qquad (2)$$

Where ε^i_t is a random variable that shows an aspect of incomplete information of the cost function for the i-firm or demand uncertainty (Dutta, 1999). The Reaction functions in matrix form is:

$$\begin{bmatrix} q^1_t \\ q^2_t \end{bmatrix} = \begin{bmatrix} \frac{a-c_1}{2b+d} \\ \frac{a-c_2}{2b+d} \end{bmatrix} + \begin{bmatrix} 0 & -\frac{b}{2b+d} \\ -\frac{b}{2b+d} & 0 \end{bmatrix} \begin{bmatrix} q^1_{t-1} \\ q^2_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon^1_t \\ \varepsilon^2_t \end{bmatrix} \qquad (3)$$

The Reaction function in Cournot's Duopoly is also expressed as:

$$\begin{bmatrix} q^1{}_t \\ q^2{}_t \end{bmatrix} = \begin{bmatrix} \omega_{11} \\ \omega_{21} \end{bmatrix} + \begin{bmatrix} 0 & -\Omega_{12} \\ -\Omega_{21} & 0 \end{bmatrix} \begin{bmatrix} q^1{}_{t-1} \\ q^2{}_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon^1{}_t \\ \varepsilon^2{}_t \end{bmatrix} \tag{4}$$

Where:

$\omega_{11} = \frac{a - c_1}{2b + d}$; $\omega_{21} = \frac{a - c_2}{2b + d}$ And $\Omega_{12} = \Omega_{21} = \frac{b}{2b + d}$

Rearranging (4), we obtain the following VAR:

$$Q_t = \omega + AQ_{t-1} + \varepsilon_t \tag{5}$$

The VAR in (5) [2] has a only equilibria point if the characteristic roots lie within a unit circle, which is in fact true as long as $\{q^1{}_t\}$ and $\{q^2{}_t\}$ are I(0). If we further assume that $q^1{}_t$ and $q^2{}_t$ are I(1), then the VAR will be represented by the following VEC:

$$\Delta Q_t = \Pi Q_{t-1} + \varepsilon_t \tag{6}$$

Where Π = - (I - A) is a 2×2 matrix. If we let r be the range of Π, then r is also the number of co-integration vectors. Thus, if r = 1 there is a one co-integration vector, and each sequence of $\{q^i{}_t\}$ can be described in terms of error corrections. By normalizing with respect to $q^1{}_t$ one obtains:

$$\begin{aligned} \Delta q^1{}_t &= \alpha_1 (q^1{}_{t-1} - \beta_1 q^2{}_{t-1}) + \varepsilon_{1t} \\ \Delta q^2{}_t &= \alpha_2 (q^1{}_{t-1} - \beta_1 q^2{}_{t-1}) + \varepsilon_{1t} \end{aligned} \tag{7}$$

In the long run, the following conditions must be met:

$$q^1{}_{t-1} - \beta_1 q^2{}_{t-1} = 0 \;;\; (q^1{}_t, q^2{}_t) \sim CI(1,1) \tag{8}$$

And [1 β_1] being the normalized co-integration vector. By the other hand, if there exists constant marginal cost, then β_1 = - ½. Observe that $\beta_1 = -\frac{b}{2b+d}$ and if d = 0 (constant marginal cost) then β_1 = - ½.

Following Johansen's method (1988, 1991, 1995), if the range (r) of matrix Π is equal to r < n (n being the number of variables), then matrix Π can be written as:

$$\Pi = \alpha\beta' \tag{9}$$

Where β' is the co-integration vector and α is the resultant vector of the adjustment velocities that assure long-term equilibrium. With regard to these adjustment velocities, we bring on the work of Fisher (1961): "…all discussions of the Cournot model allow the sellers to receive new information as to the outputs of their rivals. No seller is ever assumed to look once and for all at other outputs and then never to look again. Rather, he is assumed to look, then adjust, then look again, and so forth." Thus, the adjustment velocities tend towards equilibrium. That is, "…they are the fractions of the distance from the actual to desired output which are covered in one time period." (Fisher, 1961.) When α_i = 1, the adjustment is complete – as indicated by Theocharis (1960). Therefore, if α_i = 1 then 100% of the disequilibrium is corrected within a single period.

4- Empirical Analysis

The empirical analysis of Cournot's Model, as represented by equation (6), begins with a unit roots test for all variables. The data - from the fourth quarter of 1995 to the second quarter of 2001 - were provided by the Colombian Ministry of Communications. Since we are dealing with a series of quarters, we use the HEGY test (Hylleberg et al.,1990) to compare unit root, I(1), at zero frequency, at semiannual frequency and at annual frequency. The method is as follows:

$$\Delta_4 Y_t = \alpha + \gamma_1 Y_{1\ t-1} - \gamma_2 Y_{2\ t-1} + \gamma_3 Y_{3\ t-1} + \gamma_4 Y_{3\ t-2} + \varepsilon_t \qquad (10)$$

Where $Y_{1\,t-1} = (1+L+L^2+L^3)Y_{t-1}$, $Y_{2\,t-1} = (1-L+L^2-L^3)Y_{t-1}$, $Y_{3\,t-1} = (1-L^2)Y_{t-1}$. Test the following hypothesis:

Ho: $\gamma_1 = 0$ I(1) at zero frequency.

Ho: $\gamma_2 = 0$ I(1) at semiannual frequency.

Ho: $\gamma_3 = \gamma_4$ I(1) at annual frequency.

The results are as follow:

[Insert table 1]

From Table (1), we may conclude that the series in the Eastern Zone are integrated as to order and frequency. In the Western Zone the series are integrated at zero frequency. Additionally, the Occecc series are integrated at semiannual frequency. In the Coastal Zone, the series are integrated at different frequencies. Thus, if there is any co-integration it can only exist in the Eastern Zone. That is, only in the Eastern Zone there can be a stable relationship among the variables – and hence the possibility of long-term equilibrium – given the integration of variables as to both order and frequency, which is the requirement for co-integration[3].

Following Johansen's method, λ_{trace} was estimated so as to determine the number of co-integration vectors in the Eastern Zone [4] :

[Insert table 2]

From Table (2), we can establish the existence of a co-integration vector (r = 1) [6]. Subsequently, tests were conducted for long-term exclusion and to ascertain to what degree the variables are stationary and weakly exogenous:

[Insert table 3]

From Table (3) we conclude that no variable may be excluded unless it is both stationary and weakly exogenous.

Upon selecting the co-integration vector (r = 1), and normalizing with respect to q^{Celori}_t we obtain an estimate of (9) [7] [8] :

[Insert table 4]

From Table 4, we conclude that Celumóvil corrects 22% of the disequilibrium in its cell phone minutes within one quarter, while Comcel corrects 34%. Since at a 95% confidence level and with only 21 data the t-Statistic yields 1.72, we may conclude that the adjustment velocities are statistically significant, and that Comcel is faster when adjusting to equilibrium.

Below, we prove the hypothesis that there are constant marginal costs. From (9), we find that once we have normalized q^{Celori}_t, the hypothesis over constant marginal cost should be $\beta' =$ [1 - 0.5]. The value given by the Lr-test is 0.45, with a p-value of 0.5 [9]. Hence, at the 5% significance level, we cannot reject the hypothesis of constant marginal cost.

5- Conclusions

When one analyzes the market stability of cell phone service within the three zones into which the Colombian Government divided the market, one finds a greater level of stability in the Eastern Zone. Stability in this zone is achieved in just two or three quarters, although at times it has taken longer (as occurred between the first quarter of 1997 and the fourth quarter of 1998). The Western Zone has been increasingly unstable since 1996. As for the Coastal Zone, in the second quarter of 1998 the instability index has shown an increasing tendency to stray from zero.

The relative stability (or instability) of the market must be taken into consideration when analyzing the equilibrium conditions of each zone. In fact, the only zone displaying equilibrium between the competing firms is also the zone with greatest market stability – the Eastern Zone. Since market stability (or instability) results from the behavior of individual firms, the connection with equilibrium in Cournot's Model is intuitively logical.

In particular, the results of Cournot's Model, as applied to the Eastern Zone, indicate that the Model should take the form of an Vector Error Correction. This is because there is a co-integration vector present and the adjustment velocities are statistically significant. The results also indicate that the VEC may be restricted by (1,- 0.5) and show the existence of the constant marginal cost. The results also indicate that while Celumóvil corrects 22% of the disequilibrium in its cell phone minutes within one quarter, Comcel corrects 34%. This result is of importance to us, since the statistical significance of the adjustment velocities – being a stabilizing factor – reflect constant marginal costs within the cell phone service providers of the Eastern Zone.

In terms of welfare, the existence of equilibrium in Cournot's Model implies that cell phone customers in the Eastern Zone enjoy a smaller consumer's surplus. In a Cournot Duopoly, with positive costs for both firms and inverse demand functions $p`(Q) < 0 \; \forall \; Q>0$, the market price is above the perfect competition price, yet lower than a monopoly price (Mas-Colell et al., 1995).

Acknowledgments

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6- Appendix

1- Cournot's Model in VEC

Given the following VAR:

$$\begin{bmatrix} q^1{}_t \\ q^2{}_t \end{bmatrix} = \begin{bmatrix} \omega_{11} \\ \omega_{21} \end{bmatrix} + \begin{bmatrix} 0 & -\Omega_{12} \\ -\Omega_{21} & 0 \end{bmatrix} \begin{bmatrix} q^1{}_{t-1} \\ q^2{}_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon^1{}_t \\ \varepsilon^2{}_t \end{bmatrix} \qquad (11)$$

Assume furthermore that $\{q^1{}_t\}$ and $\{q^2{}_t\}$ are I(1). Upon subtracting Q_{t-1} from both sides of (11), we obtain:

$$\begin{bmatrix} q^1{}_t - q^1{}_{t-1} \\ q^2{}_t - q^2{}_{t-1} \end{bmatrix} = \begin{bmatrix} \omega_{11} \\ \omega_{21} \end{bmatrix} + \begin{bmatrix} 0 & -\Omega_{12} \\ -\Omega_{21} & 0 \end{bmatrix} \begin{bmatrix} q^1{}_{t-1} \\ q^2{}_{t-1} \end{bmatrix} - \begin{bmatrix} q^1{}_{t-1} \\ q^2{}_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon^1{}_t \\ \varepsilon^2{}_t \end{bmatrix} \qquad (12)$$

Rewriting (12), we obtain the following VEC:

$$\begin{bmatrix} \Delta q^1{}_{t-1} \\ \Delta q^2{}_{t-1} \end{bmatrix} = \begin{bmatrix} \omega_{11} \\ \omega_{21} \end{bmatrix} + \begin{bmatrix} -1 & -\Omega_{12} \\ -\Omega_{21} & -1 \end{bmatrix} \begin{bmatrix} q^1{}_{t-1} \\ q^2{}_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon^1{}_t \\ \varepsilon^2{}_t \end{bmatrix} \qquad (13)$$

Which is equivalent to (6). The characteristic roots of the system are:

$$\lambda_1, \lambda_2 = \frac{-2 \pm \sqrt{4\Omega_{12}\Omega_{21}}}{2} \quad \lambda_1 = -1 + \sqrt{\Omega_{12}\Omega_{21}} \quad \lambda_2 = -1 - \sqrt{\Omega_{12}\Omega_{21}}$$

If $|\lambda_1| > 1$ ó $|\lambda_2| > 1$ the process is explosive. For $\{q^1{}_t\}$ and $\{q^2{}_t\}$ to be CI(1,1), the roots must be within the unit circle (Hansen and Juselios, 1995; Davidson, 2000, 1991). As for Enders (1995, p. 239), the absolute value of one of the roots must be equal to one, and the other must be less than one. That is, $|\lambda_1| = 1$ y $|\lambda_2| < 1$.

2 – Multivariate and univariate Test

MULTIVARIATE STATISTICS

LOG(DET(SIGMA))	=	65.88923
INFORMATION CRITERIA: SC	=	67.48398
HQ	=	67.05559
TRACE CORRELATION	=	0.30847

TEST FOR AUTOCORRELATION

L-B(5), CHISQ(18) = 33.832, p-val = 0.01
LM(1), CHISQ(4) = 3.647, p-val = 0.46
LM(4), CHISQ(4) = 4.852, p-val = 0.30

TEST FOR NORMALITY

CHISQ(4) = 6.556, p-val = 0.16

ARCH(1)	Normality	R-squared
1.490	0.304	0.255
0.018	6.860	0.426

The estimated eigenvalues of A are the reciprocal values of the roots of the characteristic polynomial. Eigenvalues outside the unit disc correspond to explosive processes. The eigenvalues of the companion matrix are:

real	complex	modulus	argument
1.0000	0.0000	1.0000	0.0000
0.9786	0.0000	0.9786	0.0000

[Insert figure 2]

7- References

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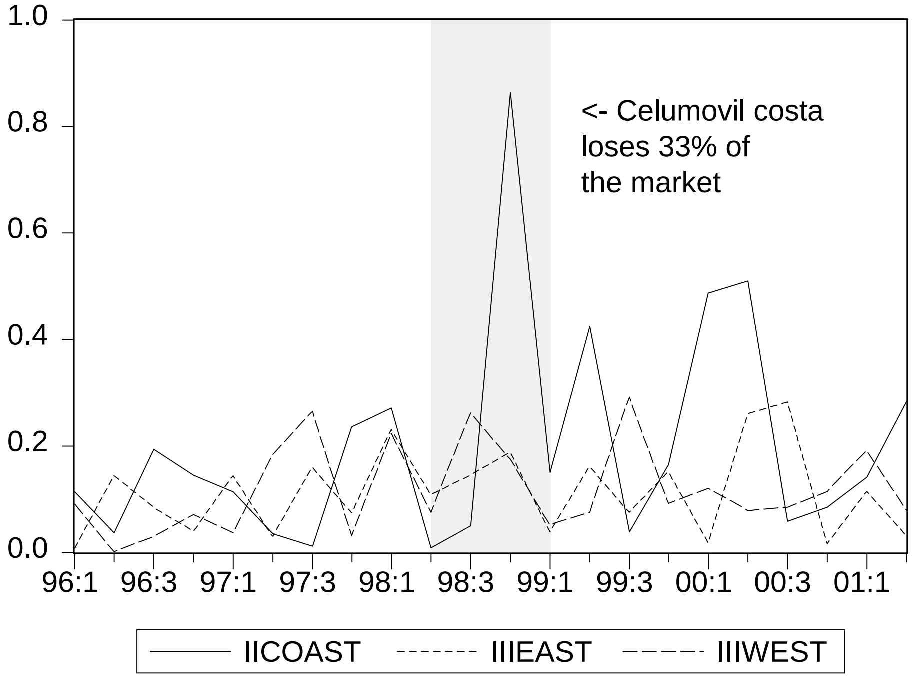
Figure 1



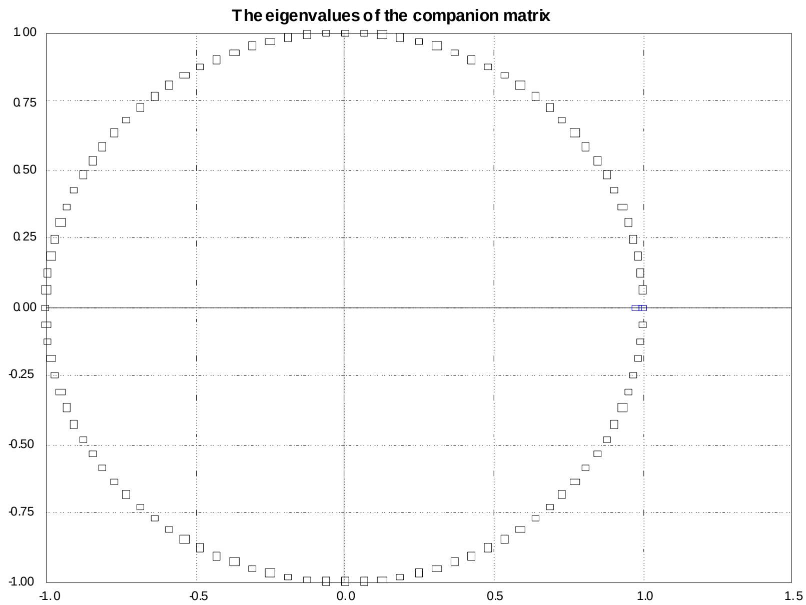


figure 2

TABLES

Table 1. Unit root test, N = 23.

Variables / γ_i		γ_1	γ_2	$\gamma_3 = \gamma_4$	Conclusion
Celori	(Celumovil Oriental)	-2.1329	2.93609	6.2416	I(1) at zero-frequency
Comori	(Comcel)	-1.04668	3.18738	4.45	I(1) at zero-frequency
Celocc	(Celumovil Occidente)	-0.54801	2.25504	5.6774	I(1) at zero-frequency
Occecc	(Occel)	0.02761	1.49434	6.63954	I(1) at zero and semiannual frequency
Celcosta	(Celumovil Costa)	-1.28600	3.14013	4.24616	I(1) at zero-frequency
Caribecosta	(Celcaribe)	-3.20664	1.56834	8.06550	I(1) at semiannual frequency
Critical Value N = 25		2.77	1.76	4.0	
Critical Value N = 30		2.78	1.79	3.82	

Note: The critical values at the 95% are reported in Charemza and Deadman (1997) table 8.

Table 2. Number of cointegrating vectors, N = 21.

Null Hypothesis	Alternative Hypothesis	λ_{trace}	90 %	95 % [5]	Conclusion
r = 0	r > 0	16.66	13.308	15.340	Reject the Null
$r \leq 1$	r > 1	2.78	2.706	3.841	Don't reject the Null

Note: Critical Values from Hansen and Juselios (1995).

Table 3. Long-run exclusion, Stationarity and Weak Exogeneity Test.

r	g.l.	$\chi^2_{95\%}(1)$	Celori	Comori	Test	Conclusion
Long- run exclusion						
1	1	3.84	9.06	10.40	Ho:β_{i1}= 0	Rejects the hypothesis of the Long-run exclusion
Stationarity						
1	1	3.84	10.40	9.06	Ho:$\beta = (H_i, \varphi)$	Rejects the hypothesis of the Stationarity
Weak Exogeneity						
1	1	3.84	4.05	7.61	Ho:α_{i1} = 0	Rejects the hypothesis of the Weak Exogeneity

Table 4. Estimated cointegrating vector β and the adjustment velocities α.

Variables	Lag	$\beta' = [\ \beta_0$	$\beta_1\ ]$	$\alpha' = [\ \alpha_{Celori}$	$\alpha_{Comori}\ ]$
$\{q^{Celori}_t, q^{Comori}_t\}$	1	1	- 0.579	- 0.221	- 0.345
				(- 2.339)	(- 3.422)

Note: t-Statistic in parentheses.

FOOTNOTES

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[1] As of March 15, 2001, Celumóvil and Cocelco changed their names to BellSouth, following the purchase of these firms by BellSouth in 2000.

[2] Given that $E(\varepsilon_t)=0$, system (5) is similar to equation (2.6) of Fisher (1961).

[3] There can be no VEC in the Coastal Zone, since the series are not I(1) at the same frequency (Hylleberg et al. (1990)). In the Western Zone, given that Celocc is I(1) at zero frequency and Occecc is I(1) at both zero and semiannual frequency, one could have co-integration. Hylleberg et al. (1990) suggest applied filters just to remove the seasonal roots, and then checking for co-integration. When this is done, with X-11 filter, the Lr-test produces 14.243 and 0.050. Thus, under no circumstances can Ho be rejected, for the series are not co-integrated. However, Otero and Smith (1999) show that seasonal filters reduce the power of the co-integration test. Nonetheless, once VEC is calculated without seasonal adjustment to the series, the adjustment velocities are not significant.

[4] For a detailed analysis of Johansen's method, see the CATS in RATS manual by Hansen and Juselios (1995), Charemza and Deadman (1997), Enders (1995), Davidson (2000).

[5] The corrections of Cheung and Lai (1993) with regard to critical values for r = 0 at 95% and 90% confidence levels are respectively 16.79 and 14.70. This, however, is not the case. "…Proper corrections of the critical values in finite samples are therefore particularly essential when the estimated system contains many variables and/or long lags," as the aforementioned authors themselves point out.

[6] Note that when r = 2, all the variables in the VAR are I(0). This contradicts the results of Table 1(Charemza and Deadman, 1997).

[7] The VEC was estimated by means of the CATS drift option. Additionally, VEC-centered seasonal dummies were employed.

[8] The tests indicate that the model is free of problems. That is, there are neither auto-correlation nor ARCH effects. Furthermore, residual values are normal and the given values are within the unit circle (see appendix).

[9] The test is of type Ho: $\beta = H\varphi$. The test employs the statistical Lr-test, which follows a χ^2 (1) distribution.