

# Cumulative Distribution Function (CDF) Analysis

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August 16, 2016

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## 1 Introduction

This document presents cumulative distribution function (CDF) analysis of a GRTS survey design. The resource used in the analysis is small lakes in Florida, which is a finite resource. Lake basins are used to delineate basins among the lakes in the resource. The analysis will include calculation of CDF estimates and testing for difference between CDFs from subpopulations of the resource.

## 2 Preliminaries

The initial step is to use the library function to load the spsurvey package. After the package is loaded, a message is printed to the R console indicating that the spsurvey package was loaded successfully.

Load the spsurvey package

```
> # Load the spsurvey package
> library(spsurvey)
>
```

Version 3.2 of the spsurvey package was loaded successfully.

### 3 Load the survey design and analytical variables data set

The original Florida small lakes data file contains more than 3,800 records and 29 basins. To produce a more manageable number of records, only six basins were retained in the data that will be analyzed, which produced a file containing 930 records.

The next step is to load the data set, which includes both survey design variables and analytical variables. The data function is used to load the data set and assign it to a data frame named FL\_lakes. The nrow function is used to determine the number of rows in the FL\_lakes data frame, and the resulting value is assigned to an object named nr. Finally, the initial six lines and the final six lines in the FL\_lakes data frame are printed using the head and tail functions, respectively.

Load the survey design and analytical variables data set

```
> # Load the data set and determine the number of rows in the data frame
> data(FL_lakes)
> nr <- nrow(FL_lakes)
>
```

Display the initial six lines in the data file.

```
> # Display the initial six lines in the data file
> head(FL_lakes)
```

	siteID	xcoord	ycoord	wgt	Basin	Status	TNT
1	FLW03414-0014	8635535	12860896	5.369048	NWFWMD-1	Sampled	Target
2	FLW03414-0046	8636136	12886783	5.369048	NWFWMD-1	Physical_Barrier	Target
3	FLW03414-0062	8617834	12869126	5.369048	NWFWMD-1	NonTarget	NonTarget
4	FLW03414-0078	8673500	12883071	5.369048	NWFWMD-1	Physical_Barrier	Target
5	FLW03414-0086	8631884	12816428	5.369048	NWFWMD-1	NonTarget	NonTarget
6	FLW03414-0118	8607699	12856644	5.369048	NWFWMD-1	NonTarget	NonTarget
	pH_Cat	Coliform_Cat	Oxygen	Turbidity			
1	(0,6]	(0,5]	9.9	0.4			
2	<NA>	<NA>	NA	NA			

```

3  <NA>          <NA>      NA      NA
4  <NA>          <NA>      NA      NA
5  <NA>          <NA>      NA      NA
6  <NA>          <NA>      NA      NA

```

```
>
```

Display the final six lines in the data file.

```

> # Display the final six lines in the data file
> tail(FL_lakes)

```

	siteID	xcoord	ycoord	wgt	Basin	Status	TNT
925	FLW03414-3878	8880656	12694963	4.80791	SWFWMD-4	Dry	Target
926	FLW03414-3886	8892406	12732977	4.80791	SWFWMD-4	Sampled	Target
927	FLW03414-3894	8836528	12723056	4.80791	SWFWMD-4	Dry	Target
928	FLW03414-3918	8923107	12725502	4.80791	SWFWMD-4	Landowner_Denial	Target
929	FLW03414-3926	8861298	12715824	4.80791	SWFWMD-4	Dry	Target
930	FLW03414-3950	8888601	12715641	4.80791	SWFWMD-4	NonTarget	NonTarget

	pH_Cat	Coliform_Cat	Oxygen	Turbidity
925	<NA>	<NA>	NA	NA
926	(6,8]	(5,50]	1.98	8.2
927	<NA>	<NA>	NA	NA
928	<NA>	<NA>	NA	NA
929	<NA>	<NA>	NA	NA
930	<NA>	<NA>	NA	NA

```
>
```

The sample of small lakes in Florida is displayed in Figure 1. The sample sites for each basin are displayed using a unique color.

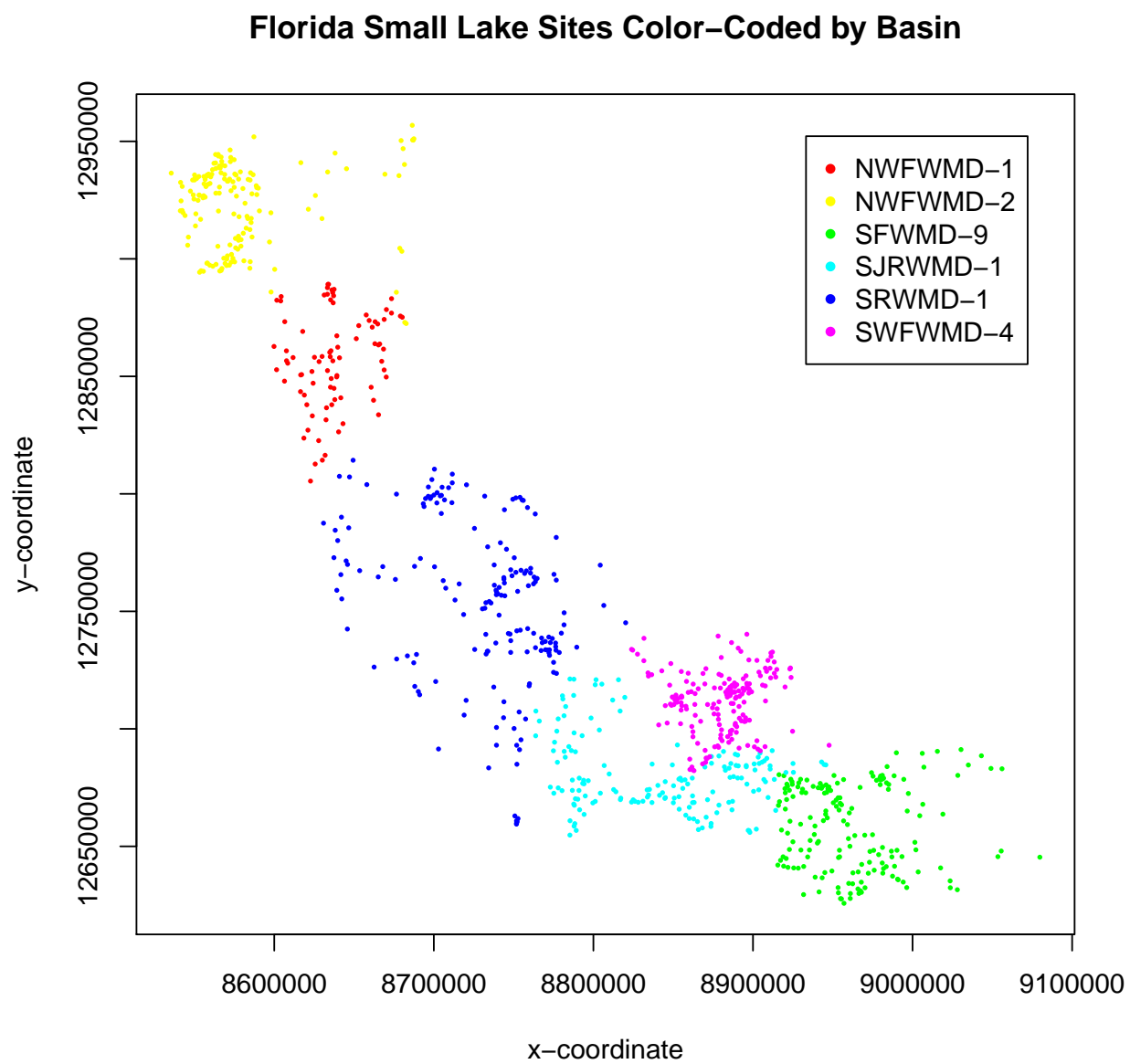


Figure 1: Location of small lake sample sites in Florida color-coded by basin.

## 4 CDF analysis of the dissolved oxygen variable

CDF analysis will be investigated by examining the dissolved oxygen variable. The summary function is used to summarize the data structure of the dissolved oxygen values.

```
> summary(FL_lakes$Oxygen)
```

Summarize the data structure of the dissolved oxygen variable:

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.	NA's
0.830	4.880	6.870	6.468	8.310	12.480	759

Note that there is an extensive number of missing (NA) values. The `cont.analysis` function will be used to calculate CDF estimates. Four data frames constitute the primary input to the `cont.analysis` function. The first column (variable) in the four data frames provides the unique identifier (site ID) for each sample site and is used to connect records among the data frames. The `siteID` variable in the `FL_lakes` data frame is assigned to the `siteID` variable in the data frames. The four data frames that will be created are named as follows: `sites`, `subpop`, `design`, and `data.cont`. The `sites` data frame identifies sites to use in the analysis and contains two variables: (1) `siteID` - site ID values and (2) `Use` - a logical vector indicating which sites to use in the analysis. The `Use` logical variables in `sites` is set equal to the value "Sampled" of the `status` variable, so that only sampled sites are used in the analysis. The `subpop` data frame defines populations and, optionally, subpopulations for which estimates are desired. Unlike the `sites` and `design` data frames, the `subpop` data frame can contain an arbitrary number of columns. The first variable in the `subpop` data frame identifies site ID values and each subsequent variable identifies a type of population, where the variable name is used to identify type. A type variable identifies each site with a character value. If the number of unique values for a type variable is greater than one, then the set of values represent subpopulations of that type. When a type variable consists of a single unique value, then the type does not contain subpopulations. For this analysis, the `subpop` data frame contains two variables: (1) `siteID` - site ID values and (2) `Basin` - which will be used to calculate estimates for each basin individually. The `basin` variable in the `FL_lakes` data frame is assigned to the `Basin` variable in the `subpop` data frame. The `design` data frame consists of survey design variables. For the analysis under consideration, the `design` data frame contains the following variables: (1) `siteID` - site ID values; (2) `wgt` - final, adjusted, survey design weights; (3) `xcoord` - x-coordinates for location; and (4) `ycoord` - y-coordinates for location. The `wgt`, `xcoord`, and `ycoord` variables in the `design` data frame are assigned values using variables with the same names in the `FL_lakes` data frame. Like the `subpop` data frame, the `data.cont` data frame can contain an arbitrary number of columns. The first variable in the `data.cont` data frame identifies site ID values and each subsequent variable identifies a response variable. For this analysis, the response variable is `DissolvedOxygen`, which is assigned variable `oxygen` in the `FL_lakes` data frame.

Create the `sites` data frame.

```
> sites <- data.frame(siteID=FL_lakes$siteID,
+                      Use=FL_lakes$Status == "Sampled")
```

Create the subpop data frame.

```
> subpop <- data.frame(siteID=FL_lakes$siteID,
+                      Basin=FL_lakes$Basin)
```

Create the design data frame.

```
> design <- data.frame(siteID=FL_lakes$siteID,
+                      wgt=FL_lakes$wgt,
+                      xcoord=FL_lakes$xcoord,
+                      ycoord=FL_lakes$ycoord)
```

Create the data.cont data frame.

```
> data.cont <- data.frame(siteID=FL_lakes$siteID,
+                          DissolvedOxygen=FL_lakes$Oxygen)
```

The frame is a data structure containing spatial location data in addition to other attributes regarding a resource of interest and is used to create a survey design. A frame often takes the form of a shapefile. The frame can be used to obtain size values (e.g., number of lakes) for the populations and subpopulations examined in an analysis. The popsize (population size) argument to `cont.analysis` provides a mechanism for forcing the size estimates of the number of lakes for each basin to sum to a desired value, i.e., the known number of lakes in each basin. As a first step for use of the popsize argument, the `c` (combine) function is used to create a named vector of frame size values for each basin. Output from the `c` function is assigned to an object named `framesize`. The popsize argument is a list, which is a particular type of R object. The popsize list must include an entry for each population type included in the subpop data frame, i.e., Basin for this analysis. Recall that the Basin population type contains subpopulations, i.e., basins. When a population type contains subpopulations, the entry in the popsize list also is a list. The `as.list` function is applied to `framesize`, and the result is assigned to the Basin entry in the popsize list. The popsize argument is included in the call to `cont.analysis`.

Assign frame size values.

```
> framesize <- c("NFWMD-1"=451, "NFWMD-2"=394, "SFWMD-9"=834, "SJRWMD-1"=1216,
+               "SRWMD-1"=1400, "SWFWMD-4"=851)
```

Use the `cont.analysis` function to calculate CDF estimates for the quantitative variables.

```
> CDF_Estimates <- cont.analysis(sites, subpop, design, data.cont,
+                               popsize=list(Basin=as.list(framesize)))
```

The object produced by `cont.analysis` is a list containing two objects: (1) `CDF`, a data frame containing the CDF estimates and (2) `Pct`, a data frame containing percentile estimates plus estimates of population values for mean, variance, and standard deviation. The `CDF` data frame contains thirteen columns. The first five columns identify the population (`Type`), subpopulation (`Subpopulation`), response variable (`Indicator`), values at which the CDF was estimated (`Value`), and number of response variable values for each value at which the CDF was estimated (`NResp`). The next four columns in the `CDF` data frame provide results for the proportion estimates, i.e., the CDF expressed as percentage values: the proportion estimate (`Estimate.P`), standard error of the estimate (`StdError.P`), lower confidence bound (`LCB95Pct.P`), and upper confidence bound (`UCB95Pct.P`). Argument `conf` for `cont.analysis` allows control of the confidence bound level. The default value for `conf` is 95, hence the column names for confidence bounds contain the value 95. Supplying a different value to the `conf` argument will be reflected in the confidence bound names. Confidence bounds are obtained using the standard error and the Normal distribution multiplier corresponding to the confidence level. The final four columns in the data frame provide results for the size (units) estimates, i.e., the CDF expressed in terms of number of lakes: the units estimate (`Estimate.U`), standard error of the estimate (`StdError.U`), lower confidence bound (`LCB95Pct.U`), and upper confidence bound (`UCB95Pct.U`).

Unlike the `CDF` data frame, the `Pct` data frame contains only nine columns since there is a single set of estimates rather than two sets of estimates. In addition, the fourth column is labeled `Statistic` and identifies either a percentile or the mean, variance, or standard deviation estimate. Finally, since percentile estimates are obtained by inverting the CDF estimate, the percentile estimates do not have a standard error value associated with them.

Use the `write.csv` function to write the CDF estimates as a csv file.

```
> write.csv(CDF_Estimates$CDF, file="CDF_Estimates.csv", row.names=FALSE)
```

The `cont.cdfplot` function in `spsurvey` can be used to produce a PDF file containing plots of the CDF estimates. The primary arguments to `cont.cdfplot` are a character string containing a name for the PDF file and the `CDF` data frame in the `CDF_Estimates` object.

Produce a PDF file containing plots of the CDF estimates.

```
> cont.cdfplot("CDF_Estimates.pdf", CDF_Estimates$CDF)
>
```

The `cont.cdfctest` function in `spsurvey` can be used to test for statistical difference between the CDFs from subpopulations. For this analysis we will test for statistical difference between the CDFs from the six basins. The `cont.cdfctest` function will test all possible pairs of basins. Arguments to `cont.cdfctest` are the same as arguments to `cont.analysis`.

```
> CDF_Tests <- cont.cdfctest(sites, subpop, design, data.cont,
+   popsize=list(Basin=as.list(framesize)))
```

The print function is used to display results for dissolved oxygen of the statistical tests for difference between CDFs for basins. The object produced by `cont.cdftest` is a data frame containing eight columns. The first column (`Type`) identifies the population. The second and third columns (`Subpopulation_1` and `Subpopulation_2`) identify the subpopulations. The fourth column (`Indicator`) identifies the response variable. Column five contains values of the test statistic. Six test statistics are available, and the default statistic is an F-distribution version of the Wald statistic, which is identified in the data frame as "Wald-F". The default statistic is used in this analysis. For further information about the test statistics see the Appendix. Columns six and seven (`Degrees_of_Freedom_1` and `Degrees_of_Freedom_2`) provide the numerator and denominator degrees of freedom for the Wald test. The final column (`p_Value`) provides the p-value for the test.

```
> # Print results of the statistical tests for difference between CDFs from
> # basins for dissolved oxygen
> print(CDF_Tests, digits=3)
```

	Type	Subpopulation_1	Subpopulation_2	Indicator	Wald_F
1	Basin	NFWWMD-1	NFWWMD-2	DissolvedOxygen	3.144
2	Basin	NFWWMD-1	SFWMD-9	DissolvedOxygen	4.480
3	Basin	NFWWMD-1	SJRWMD-1	DissolvedOxygen	20.292
4	Basin	NFWWMD-1	SRWMD-1	DissolvedOxygen	0.305
5	Basin	NFWWMD-1	SWFWMD-4	DissolvedOxygen	10.668
6	Basin	NFWWMD-2	SFWMD-9	DissolvedOxygen	2.609
7	Basin	NFWWMD-2	SJRWMD-1	DissolvedOxygen	6.161
8	Basin	NFWWMD-2	SRWMD-1	DissolvedOxygen	2.819
9	Basin	NFWWMD-2	SWFWMD-4	DissolvedOxygen	3.822
10	Basin	SFWMD-9	SJRWMD-1	DissolvedOxygen	12.760
11	Basin	SFWMD-9	SRWMD-1	DissolvedOxygen	6.088
12	Basin	SFWMD-9	SWFWMD-4	DissolvedOxygen	14.118
13	Basin	SJRWMD-1	SRWMD-1	DissolvedOxygen	16.973
14	Basin	SJRWMD-1	SWFWMD-4	DissolvedOxygen	5.237
15	Basin	SRWMD-1	SWFWMD-4	DissolvedOxygen	6.409
	Degrees_of_Freedom_1	Degrees_of_Freedom_2	p_Value		
1		2	55	5.09e-02	
2		2	57	1.56e-02	
3		2	57	2.21e-07	
4		2	54	7.39e-01	
5		2	51	1.35e-04	
6		2	55	8.27e-02	
7		2	55	3.85e-03	
8		2	52	6.88e-02	
9		2	49	2.87e-02	
10		2	57	2.63e-05	
11		2	54	4.13e-03	
12		2	51	1.32e-05	
13		2	54	1.91e-06	
14		2	51	8.54e-03	
15		2	48	3.41e-03	

```
>
```

Since there is a fairly large number of CDFs being compared (15), it is reasonable to use a conservative p-value for assessing whether a pair of CDFs is significantly different. We will use a p-value no larger than 0.01 to test for significant difference. Using that value, basins with significantly different CDFs are displayed in Figure 2 using "X" to indicate CDFs that are different. Note that only unique pairs of basins are displayed in the Figure 2, which is reflected in the figure by the red line. Significantly different CDFs for pairs of basins located to the right of the red line are displayed in the figure. Nine pairs of CDFs are different: (1) basin NFWWMD-1 is different from basins SJRWMD-1 and SWFWMD-4; (2) basin NFWWMD-2 is different from basin SJRWMD-1; (3) SFWMD-9 is different from basins SJRWMD-1, SRWMD-1, and SWFWMD-4; (4) SJRWMD-1 is different from basins SRWMD-1 and SWFWMD-4; and (5) SRWMD-1 is different from basin SWFWMD-4. The majority of the significantly different CDFs are attributable to two basins. The CDF for basin SJRWMD-1 is different from the CDFs for all of the other basins. The CDF for basin SWFWMD-4 is different from the CDFs for all of the other basins except basin NFWWMD-2. The only other pair of basins with different CDFs are basins SFWMD-9 and SRWMD-1.

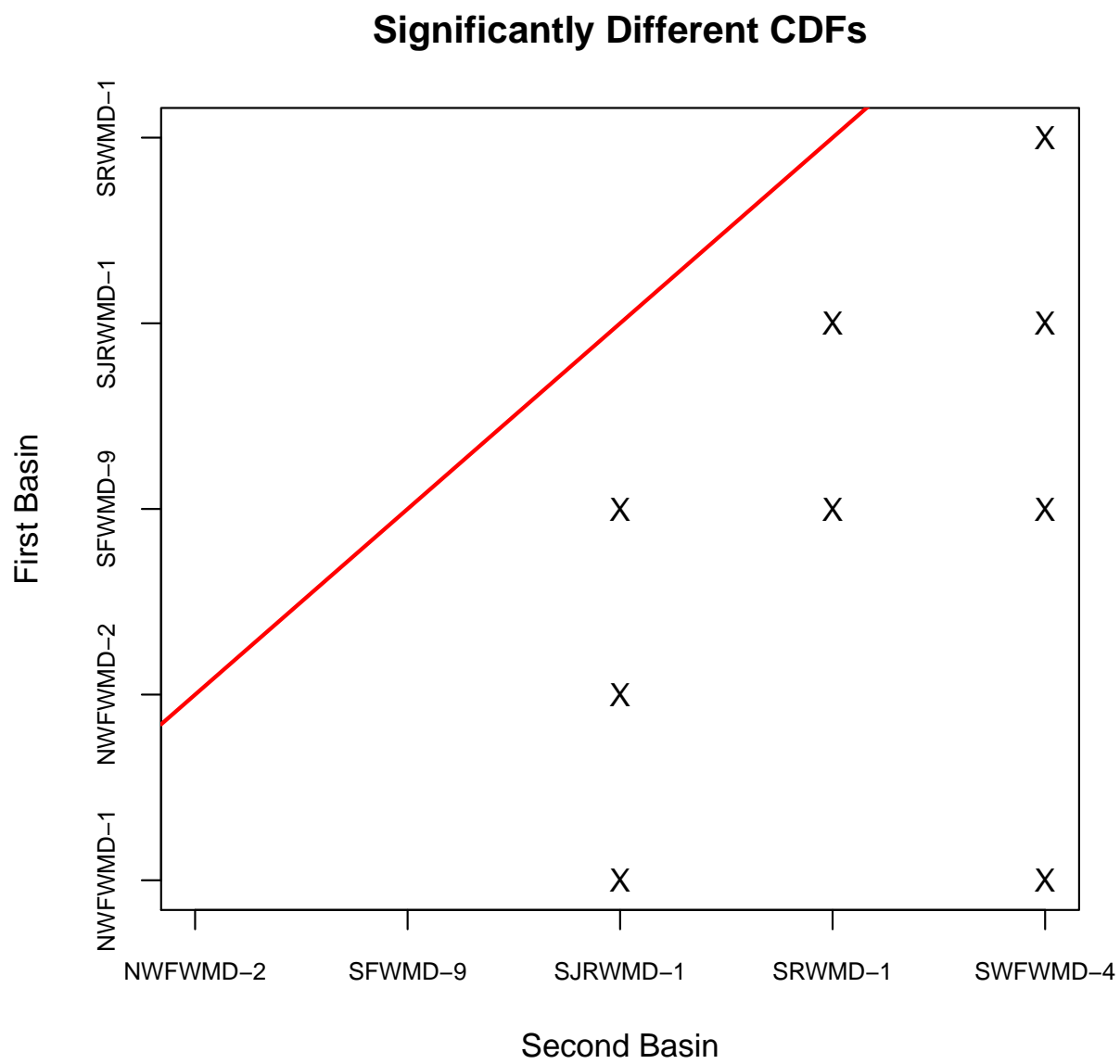


Figure 2: Florida Basins with Significantly Different CDFs.

Use the `write.csv` function to write CDF test results as a csv file.

```
> # Write CDF test results as a csv file  
> write.csv(CDF_Tests, file="CDF_Tests.csv", row.names=FALSE)  
>
```

## A Appendix – Inferential Procedure

The inferential procedure employed to test for differences between CDFs partitions the two CDFs that are being tested into a discrete set of intervals (classes) and then utilizes procedures that have been developed for analysis of categorical data from probability surveys (Kincaid 2000). The `cont.cdfctest` function calculates the Wald, Rao-Scott first order corrected (mean eigenvalue corrected), and Rao-Scott second order corrected (Satterthwaite corrected) test statistics (Rao and Scott 1981; Wald 1943). Both standard versions of the three statistics, which are distributed as Chi-squared random variables (Rao and Thomas 1988), and alternate version of the statistics, which are distributed as F random variables (Thomas, Roberts, and Singh 1996), are available as options in the `cont.cdfctest` function. The Horvitz-Thompson ratio estimator, i.e., the ratio of two Horvitz-Thompson estimators (Horvitz and Thompson 1952), is used to calculate estimates of the class proportions for the CDFs. Variance estimates for the test statistics are calculated using either the local mean variance estimator or the simple random sampling (SRS) variance estimator. The choice of variance estimator is subject to user control. The SRS variance estimator uses the independent random sample approximation to calculate joint inclusion probabilities.

## References

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